A Universal Power Transfer Curve

R. C. Hansen, *Life Fellow, IEEE*, James McSpadden, *Member, IEEE*, and James N. Benford, *Fellow, IEEE*

***Abstract*—Power transfer between circular apertures at far-field distances is calculated using a widely applicable aperture distribution with adjustable sidelobe ratio. Power transfer efficiency is obtained by integrating the pattern out to the angle subtended by the receiving aperture. Efficiency is parameterized such that a single curve results for each sidelobe ratio, independent of transmit D/λ, over a range of 5 to 500.**

***Index Terms*—One-parameter circular distribution, wireless power transfer.**

I. Introduction

WIRELESS transfer of power from one antenna to another is again of interest, following early proposals that date back to the 1960's. A variety of schemes have been suggested for transferring energy from earth-to-space, space-to-earth, space-to-space, and between points on earth using high power microwave beams. Possible applications include solar power to earth, satellite to other space vehicle, earth to UAV, etc. [1], [2]. A current installation provides power transmission to an isolated canyon, where power pylons would mar the scenery. Of concern is far-field transfer, where the separation between antennas is at least equal to the far-field distance $2D^2/\lambda$, where D is the diameter of the largest antenna. Wavelength is λ. The method described herein can also be applied to a focused transmit antenna, with spot size less than $D_{\rm t}$; the focused pattern is used.

Previous significant work was by Goubau [3], [4]; he used the Fresnel–Kirchhoff approximation (now called physical optics); this does not adequately take into account the sidelobe envelope, nor the aperture amplitude distribution and the related Q or bandwidth. For low sidelobes, the pattern function resulting from the Goubau Gaussian aperture, has nonmonotonic zero spacing, thus violating Taylor's rules for optimum (efficient) patterns [5]. A Chebyshev type distribution, would maximize the fraction of power in the main beam, but these often have nonmonotonic amplitude distributions, and narrow bandwidth. A more useful optimum considers the sidelobe envelope, to insure a robust aperture, and to minimize interference from far-out sidelobes. Thus the Goubau formulation is not optimum. However the parameterization used by Goubau is used below, modified slightly.

Manuscript received October 21, 2004; revised February 18, 2005. This work was supported by DARPA under MDA972-03-P-0004. The review of this letter was arranged by Associate Editor M. Mrozowski.

R. C. Hansen is a Consulting Engineer in Tarzana, CA 91357 USA (e-mail: www.rchansen.com).

J. McSpadden is with McSpadden Consulting, Seattle, WA 98146 USA.

J. N. Benford is with the Microwave Sciences, Lafayette, CA 94549 USA.

Digital Object Identifier 10.1109/LMWC.2005.847718

II. Circular One-Parameter Distribution

A circular transmitting aperture is assumed, as this is the most effective area for power transmission. Taylor [5] developed aperture distributions that utilized only one parameter, and were optimum for a given sidelobe ratio (SLR) (main beam peak/first sidelobe). This concept was extended to a circular aperture [6] of diameter D. The parameter is called H, and the pattern, symmetric about its axis, is given by

$$E(\theta) = \frac{I_1(\pi\sqrt{H^2-u^2})}{\pi\sqrt{H^2-u^2}} \text{ for } H \geq u \tag{1}$$

$$E(\theta) = \frac{J_1(\pi\sqrt{u^2-H^2})}{\pi\sqrt{u^2-H^2)}} \text{ for } u \geq H \tag{2}$$

where the variable $u = D/\lambda \sin\theta$; θ is the polar angle. The pattern transitions from the I_1 Bessel function to the J_1 Bessel function, roughly in the middle of the main beam region. All sidelobes are represented by the J_1 form. Given H, all characteristics are easily found. The aperture, or array, distribution is given by

$$g(\rho) = I_o(\pi(\sqrt{1-\rho^2}). \tag{3}$$

Other parameters, such as aperture taper efficiency, main beam efficiency (null-to-null), 3-dB beamwidth, and null-to-null beamwidth, are readily determined given H.

This circular one-parameter distribution is often used for design of circular planar arrays. Using SLR of 25 dB, the aperture distribution and pattern are a good fit to those of most reflector antennas. Thus this realistic and optimum circular one-parameter distribution is used herein.

III. Calculation of Power Transfer

It is assumed that the receiving aperture has diameter W and is at distance R. The half angle subtended by the receive aperture is θ_w

$$\theta_w = \arctan\left(\frac{W}{2R}\right). \tag{4}$$

The maximum power transfer is simply

$$\eta = \frac{\int_0^{\theta_w} |E|^2 \sin\theta d\theta}{\int_0^{\pi/2} |E|^2 \sin\theta d\theta} \tag{5}$$

where E is given by (1). Clearly numerical integration is required, but that in the denominator can be related to the aperture taper efficiency $\eta_{\rm t}$

$$\eta_{\rm t} = \frac{E_o^2\lambda^2}{\pi^2 D^2 \int_o^{\pi/2} |E|^2 \sin\theta d\theta} \tag{6}$$

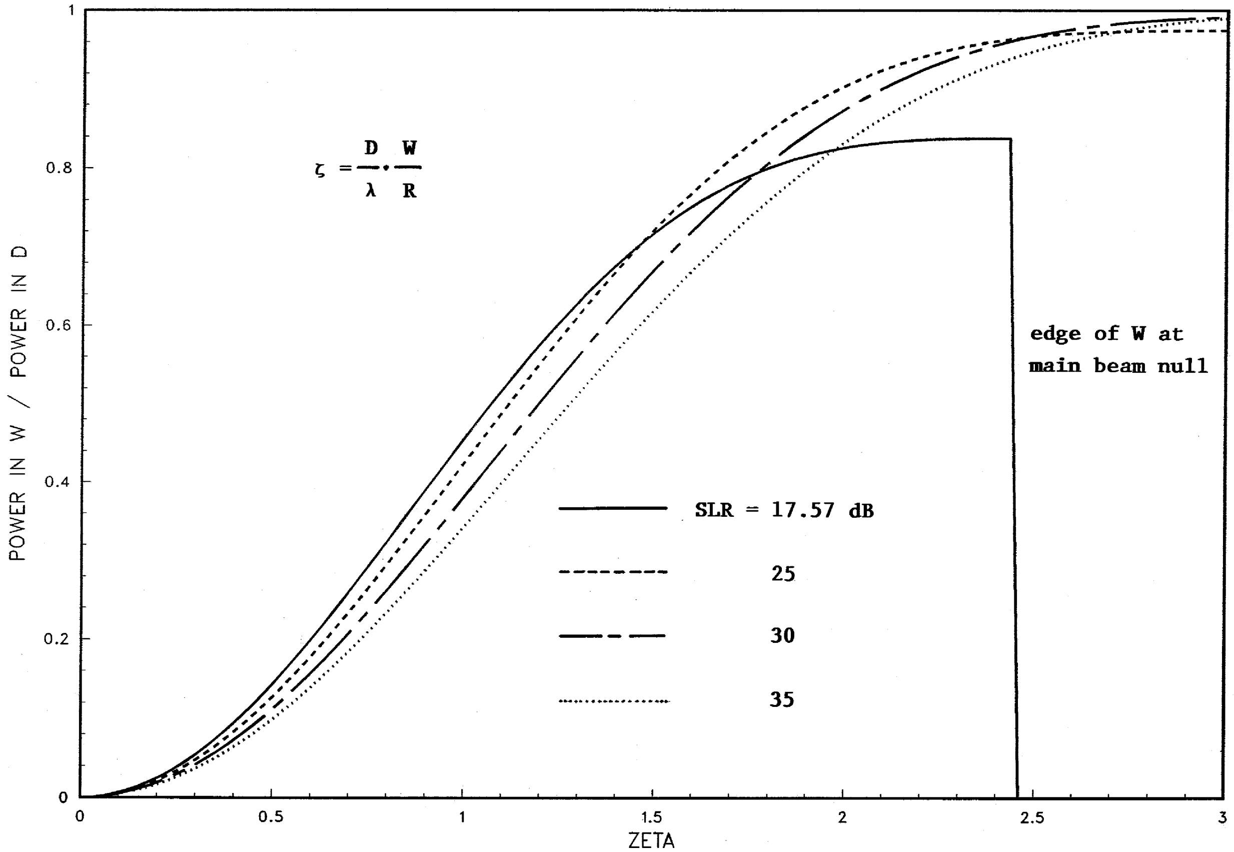

Fig. 1.Power transmission efficiency.

where E_0 is the electric field at $\theta = 0$. This taper efficiency does not require an integration as it can be expressed simply in terms of I_0 and I_1 of argument πH, and is well tabulated [6]. This allows the power transfer efficiency to include only the integration from 0 to θ_w

$$\eta = \frac{\pi^2 D^2 \eta_t}{2\lambda^2} \int_o^{\theta_w} |\frac{E}{E_o}|^2 \sin\theta d\theta. \qquad (7)$$

Sidelobe ratio is probably an easy choice, leaving η dependent upon two variables. It was a surprise to find that the parameterization of Goubau, with slight changes, produced a universal power transfer curve. The parameter ζ is

$$\zeta = \frac{DW}{\lambda R}. \qquad (8)$$

Keeping this parameter fixed, the integration of (6) produced the same value of η to 4 significant figures for ζ from 0 to 3, for D/λ varying from 10 to 500. The reason why this occurs is that most of the value of the integral results from the pattern peak to the transition angle; as H changes the beam shape changes, and the transition point moves. These two effects tend to cancel out. Similarly the shift in transition point and the sidelobe integral tend to cancel. The result is Fig. 1, which shows power transmission efficiency η for sidelobe ratios of 17.57 (uniform excitation), 25, 30, and 35 dB, versus ζ, ranging from zero to three.

The receiving aperture equals the main beam null-to-null width when

$$\zeta = \frac{\sqrt{2|\frac{Z_1}{\pi}|^2 + H^2}}{\sqrt{1 - \frac{|\frac{Z_1}{\pi}|^2 + H^2}{\left(\frac{D}{\lambda}\right)^2}}} \qquad (9)$$

where $Z_1 = 3.831\,71$ is the first zero of J_1. For SLR of 17.57 and 25 dB, this occurs for $\zeta = 2.440$ and $\zeta = 3.021$. Accordingly the uniform excitation curve has been made to drop to zero at the value, as it is undesirable to illuminate the receiving antenna with sidelobes.

In ζ, antenna diameters are used because: antenna designers usually work in diameters rather than in areas; and the beam must encircle the receiving antenna whether it is circular or not. For cases where the receiving antenna (rectenna) is not circular, η is simply decreased by the rectenna area divided by $\pi \mathrm{W}^2/4$. It is believed that this universal power transfer curve is useful, as it is optimum for each sidelobe level.

As an example of this formulation, consider the case of an aircraft at 1 km altitude receiving power for its propulsion from a ground based antenna of 4 m diameter, radiating at 12 GHz. And assume a sidelobe ratio of 25 dB. From Fig. 1 the collection efficiency will be 90% for $\mathrm{zeta} = 1.977$. This corresponds to a rectenna diameter of 12.36 m, and this diameter must encompass the rectenna. Note that these efficiencies are for circular polarization; other polarizations can lead to additional loss.

REFERENCES

[1] E. J. Nalos, “New developments in electromagnetic energy beaming,” *Proc. IEEE*, vol. 55, no. 3, pp. 276–289, Mar. 1978.

[2] W. C. Brown, “Beamed microwave power transmission and its application to space,” *Trans. IEEE Microw. Theory Tech.*, vol. 40, no. 6, pp. 1239–1250, Jun. 1992.

[3] G. Goubau and F. Schwering, “Free space beam transmission,” in *Microwave Power Engineering*, E. C. Okress, Ed. New York: Academic, 1968, vol. 1, pp. 241–255.

[4] G. Goubau, “Microwave power transmission from an orbiting solar power station,” *J. Microw. Power*, vol. 5, pp. 223–231, 1970.

[5] R. C. Hansen, *Phased Array Antennas*. New York: Wiley, 1998.

[6] ——, “A one-parameter circular aperture distribution with narrow beamwidth and low sidelobes,” *Trans. IEEE Antennas Propag.*, vol. AP-24, no. 7, pp. 477–480, Jul. 1976.