Experimental Tests Of Beam-Riding Sail Dynamics

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Abstract. Stability is a neglected issue in proposals to propel light sails by beamed power. Whether the beam comes from a laser or a microwave antenna, power falls with angle from the beam center. This drives a sail sideways under any lateral perturbation—"tumbling down the hill.". While spin can help stabilize, the basic mechanics of pressures and sail averaging of them across its area remain unexplored in experiment, and only recently treated in theory. Here we report what is to our knowledge the first attempt to study beam-riding dynamics in the laboratory, using a slightly overweighted pendulum. A sail attached to the pendulum bottom can be made unstable by adding weight to the top end. Stability and oscillation are possible if this is corrected by electrodynamic beam pressure on the sail, directed from below, torquing the pendulum. We present both data and analysis. Our major points are: Microwave powers of a few hundred W can hold a sail steady. This is made possible because of the gradient in beam power with sidewise angle. Our experiments agree with the Univ. New Mexico numerical studies which show similar stability conditions. At higher powers, the sail can be oscillated in angle. Time-dependent feedback of beam power can manage a sail into stable motions across the beam. Theory shows this, but experiments are not yet done. Beam powers comparable to the strength of perturbing forces can plausibly achieve these effects in free sail flight.

INTRODUCTION

Generally, beam-riding sails are unstable, and will move away from the beam center in uncontrolled fashion. Here we describe experiments which explore the electrodynamic restoring forces acting on sails, which can in principle lead to stable sail flight.

To study stability in a g-field, one must compensate for gravity and yet give a source of instability. This implies the schematic design of Fig 1, where a sail hangs

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from a rod that pivots about a bearing. The sail has mass M^* and the rod M'. Above the bearing is a counter-weight M which provides very nearly compensating mass, $M > M^* + M'$, making the whole assembly slightly unstable. The microwave source below provides a pressure which can make the sail return to vertical, hanging straight down, i.e., the angle θ goes to zero.

An important result emerges if the sail is displaced a distance d from the beam axis. It turns out that the *gradient* of the microwave power in angle is the stabilizing element, because the *difference* in torques about the sail center drives rotation of the assembly. Moving the sail slightly aside (~ 1 cm) helps minimize the needed power. Generally, the sail will fall to the side, increasing θ, and have some momentum as it passes through the ideal angle $\sim d/H$. It will go a bit further, then be arrested, where the mechanical torque equals the microwave torque averaged over the sail.

Still, there are considerable design requirements; one must fine-tune the lengths of the rod and counter-weight, and the counterweight mass. In our simple theoretical model, power P falls with angle as $\cos^2\theta$, which helps off-center sail placement. A steeply falling beam would be much easier to stabilize. Experiment finds a gradient $\cos^m\theta$ with m ~ 5, promising ready stabilization. In a true, flying sail in zero-g, this may be crucial.

Flipping this arrangement, so the sail is upright and falls from vertical as it goes unstable, yields a *higher* power demand. This comes from geometric effects. Basically there are three geometric and physical effects affecting stability:

1. angle between wave and surface
2. area exposed
3. falloff of power with angle from the beam axis

The hanging sail mode (pendulum-like) wins overall for these factors.

Specifically, take $\theta = 12^\circ$ and $d/R = 0.1$, with a 2 cm radius sail hanging 10 cm from the microwave source, on a rod 10 cm long. With the counter-weight to be 9 mm above the pivot (bearing), then the sail will be stable to sideways tumbling at power levels of 860 W or more. Roughly a kW should show beam-riding, then. This prediction we verified.

THEORY

The pendulum in Fig.1 follows the *difference in torques* from microwaves ("rf") and gravity ("gr") about the axis, which bisects the circular sail. Let us consider a pendulum with a counter mass M_c equal to the mass of the sail $M_c = M_s$. The pivot of the pendulum is at the distance d with respect to the *center line* of the waveguide.

M_s is the mass of the sail,

R is the radius of the sail (or 2R is a side of a square),

L_o is the length of the pendulum,

$2L_o$ is the length of the rod,

ΔL is the distance from the pivot to the center of mass,

$k_o = L_o / \Delta L.$,
$H_o + L_o$ is the distance from the pivot to plane of the edge of the waveguide,
L is the distance from the waveguide to the center of the sail,
θ is the angle of microwave beam propagation (see the figure), and
ϕ_1 is the pendulum decline angle from vertical position.

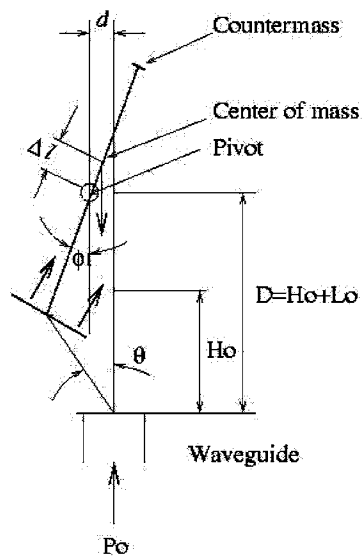


FIGURE 1. Pendulum with counter-mass off center

Static case: Pendulum with No Counter-Weight; Minimum Needed Pressure

To maintain the pendulum at an angle ϕ_1 the torque produced by the force due to the microwave field should be equal to the torque produced by the force of gravity. We consider the case of total reflection and no absorption of microwaves by the sail material. For a given ϕ_1, H_o, L_o we have

$$\theta = \arctan\left(\frac{\sin\phi_1 + d/L_\circ}{H_\circ/L_\circ + 1 - \cos\phi_1}\right).$$

$$1 = \frac{L_\circ(H_\circ/L_\circ + 1 - \cos\phi_1)}{\cos\theta}, \; \beta_1 = \phi_1 + \theta.$$

and the torque equilibrium equation is

$$T_{grav} = T_{rf}.$$

where T_{grav} is torque due to the gravitational force and T_{rf} is the torque due to microwave pressure, here:

$$T_{grav} = M_s g \Delta L \sin\phi_1 ,$$

$$T_{rf} = \frac{P_\circ A_k 2\int rf(r)dS}{c} = P_\circ A - 2R(2K/c).$$

where S is the area of the sail, r is a coordinate on the surface of the sail, A_k is a normalization coefficient (A_k = 0.38 for Bessel $J_1(\theta)$distribution of the field in the plane XOZ and $A_k = 2\pi/3$ for $\cos^2\theta$ distribution of the RF field in the plane XOZ), P_o is the microwave power in the waveguide, $f(r)$ is the function proportional to the force due to effect of microwaves, c is the speed of light. The function $f(r)$ is proportional to $J_1^2(\theta)$ or to $cos^2\ \theta$ due to the field distribution pattern and proportional to cos ^{2}(incident angle) due to the photon reflection from the sail. The K-integral assembles all the geometric factors governing electrodynamic pressures on the sail.

Then the *torque equation* is (mks units)

$$M_s g \Delta L \sin\phi_1 = P_\circ A_k 4R\,K/c \qquad (1)$$

and the microwave power in the waveguide is

$$P_\circ = \frac{M_s gcL_\circ \sin\phi_1}{4A_k KRk_\circ} = \frac{M_s gc\Delta L \sin\phi_1}{4A_k KR} \qquad (2)$$

where $g = 9.8$ m /sec^2, $c = 3\times10^8$ m /sec, $M_s = \rho \times 4R^2$, $\rho = 0.005$ kg/m^2 for our carbon sail and A_k is given above.

Dynamic case-- Pendulum with Counter-Weight

Consider a pendulum with a counter mass M_c equal to the mass of the sail $M_c = M_s$ and a pivot at a distance d with respect to the center line of the waveguide.

M_r is the mass of the rod,

$2L_o$ is the length of the rod,

The pivot point is at the center of the rod,

M_s is the mass of the sail, and

I is the moment of inertia of the system.

The equation of motion for the system is:

$$I\frac{d^2\phi_1}{dt^2} = T_{grav} - T_{rf} \qquad (3)$$

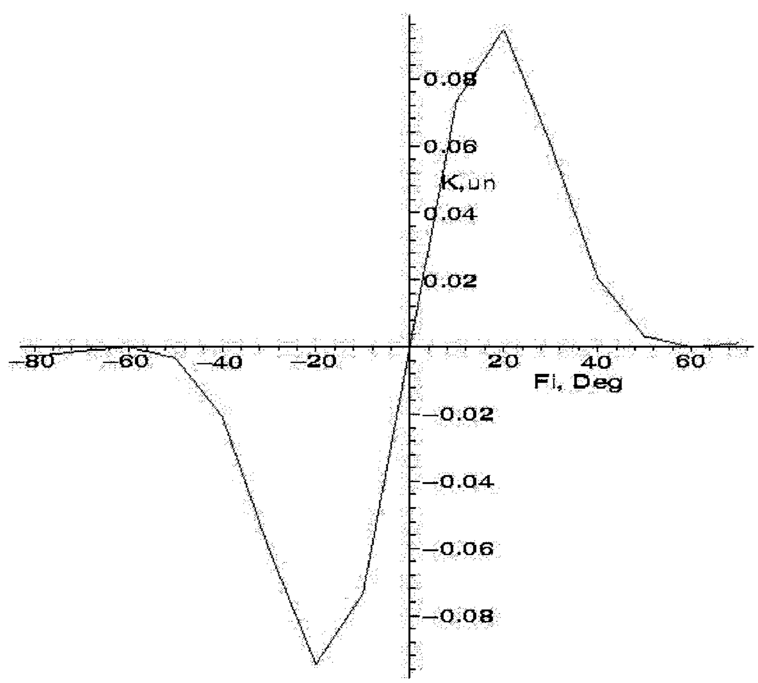
where T_{grav} is the torque due to the gravitational force and T_{rf} is the torque due to the microwave pressure. In order to solve the motion equation we have to calculate the moment of inertia I for the system considered:

$$I = M_r (2L_\circ)^2/12 + M_s L_\circ^2 + M_s (L_\circ + \Delta L)^2 \approx M_r L_\circ^2/3 + 2M_s L_\circ^2 + 2M_s L_\circ \Delta L$$
$$= M_r L_\circ (L_\circ/3 + 2M_s/(M_r \times L_\circ) + 2M_s/(M_r \times \Delta L))$$

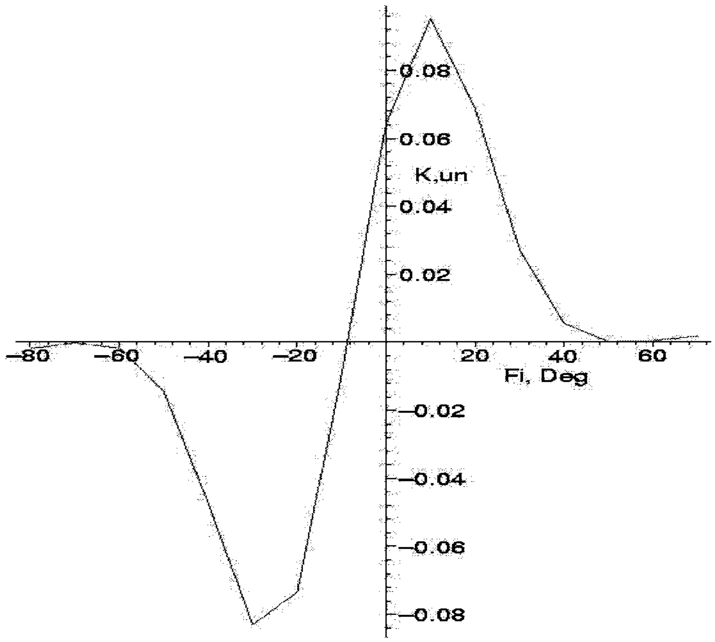
Then the equation of motion will be

$$I\frac{d^2\phi}{dt^2} = M_s g \Delta L \sin\phi_1 - P_\circ A_k 2R\frac{2K}{c} \ . \qquad (4)$$

Let us consider the value of integral K versus angle ϕ_1. The equation (1) for K gives us the direction of the torque due to the microwave pressure versus the angle of deflection ϕ_1 for given pendulum geometry. The sign of K is positive for counter clockwise rotation due to the RF torque and negative for clockwise rotation due to the RF torque.



(a)



(b)

FIGURE 2. The value of integral K versus deflection angle ϕ_1 for centered pendulum (a) and off center pendulum (b), d = 1 cm. Note the asymmetry when d is not zero; this is the source of stabilizing torques.

The plot for $K\ (\phi_1)$ is shown in Fig.2 for a given geometry of the pendulum and its position with respect to the waveguide. As can be seen, we can use a linear approximation for the value of K for angles within the range of -23 deg to 10 deg:

$$K(t) = K_\circ + K\phi_1(t)\,.$$

where $\phi_1(t)$ is in radians, $K_o = K\,(0) = 0.0643$ and k = 0.298 {1/rad].

For small angles ϕ_1 ($\phi_1 < 17$ deg) we have sin $\phi_1 \sim \phi_1$

Then we have the following equation of motion for small angles ϕ_1

$$\frac{d^2\phi_1}{dt^2} = \frac{\left(M_s g\Delta L\phi_1 - P_\circ A_k 4R\left(\frac{K_\circ + k\phi_1}{c}\right)\right)}{I} \tag{5}$$

$$= \phi_1\left(\frac{M_s g\Delta L}{I} - P_\circ A_k 4R\frac{k\phi_1}{cI}\right) - P_\circ A_k 4R\frac{K_\circ}{Ic} = A + B\phi_1$$

where

$$A = -P_\circ A_k 4R\frac{K_\circ}{Ic} \quad \text{and} \quad B = \frac{M_s g\Delta L}{I} - P_\circ A_k 4R\frac{k}{cI}.$$

In order to solve the equation analytically, perform the following transformation:

$$y(t) = A + B\phi_1(t).$$

then

$$\frac{d^2 y(t)}{dt^2} = By(t), y(0) = A + B\phi_1(0).$$

This equation can be solved analytically, with solution

$$y(t) = (1/Ic)(-P_\circ A_k 4RK_\circ - P_\circ A_k 4Rk\phi_1(0) + M_s g\Delta Lc\phi_1(0))\times \cos\left(\left(\sqrt{(P_\circ A_k 4Rk - M_s g\Delta Lc)/cI}\right)t\right)$$

$$\begin{aligned}\phi_1(t) = \{&(1/Ic)(-P_\circ A_k 4RK_\circ - P_\circ A_k 4Rk\phi_1(0) + M_s g\Delta Lc\phi_1(0)) \\ &\times \cos\left(\left(\sqrt{(P_\circ A_k 4Rk - M_s g\Delta Lc/cI)}\right)t\right) + \\ &P_\circ A_k 4RK/cI\}/(M_s g\Delta L/I - P_\circ A_k 4Rk/cI)\end{aligned} \qquad (6)$$

We can write that

$$\phi_1(t) = \phi_1(0)\cos(\omega t) + A_0(1 - \cos(\omega t)) = A_0 + (\phi_1(0) - A_0)\cos(\omega t).$$

where

$$A_0 = (K_\circ/k)(P(\omega = 0)/P_\circ - 1) - 1.$$

$$\omega = 2\pi/T = \sqrt{(P_\circ A_k 4Rk/c - M_s g\Delta L)/I}.$$

critical stabilizing power:

$$P_\circ(\omega = 0) = \frac{M_s g\Delta Lc}{A_k 4Rk}.$$

Unbounded motion:
This solution will be a cosh function for power values satisfying the inequality

$$P_\circ A_k 4Rk - M_s g\Delta Lc < 0$$

Bounded motion:
Solutions are a cos function for power values satisfying the inequality

$$P_\circ A_k 4Rk - M_s g\Delta Lc > 0$$

with angular velocity

$$\omega = 2\pi/T = \sqrt{(P_\circ A_k 4Rk/c - M_s g\Delta L)/I}$$

$$\omega = 0\text{, if } P_\circ(\omega = 0) = \frac{M_s g\Delta Lc}{A_k 4Rk}. \qquad (7)$$

As we can see, period of oscillations does not depend on initial deflection angle and depends only on the microwave power and geometrical parameters of the system.

EXPERIMENT

The microwave source is a CW 10.6 GHz klystron with maximum power 10 kW. The waveguide assembly used WR90 and waveguide components in X-band. The microwaves are emitted from a horn with narrow power distribution pattern with a gradient ~$cos^m\,\theta$ with m ~ 5.

Since the major part of the microwave power is reflected towards the bottom of the chamber, we covered the bottom of the chamber with silicon carbide tiles were used as an absorber. We used a 2.5-cm radius disk sail made of carbon fiber microtruss manufactured by Energy Science Laboratories Inc. A 3-cm long axle was placed through the center hole of an aluminum tube of 8-cm length and fixed with torr seal. A disk made of aluminum foil or a nut made of the nylon was used as a counterweight in the experiments. The sail-aluminum rod connection was made using thin silver wire. A digital video camera views the pendulum inside the experimental chamber through the glass window and hexcell screen in the viewing port.

We placed the same sail at the distance of 4 cm from the open end of the waveguide and off-center 1 cm from the center line of the waveguide (d = 1 cm in Figure 1). The initial angle of deflection from the position of unstable equilibrium was varied from 2 to 5 degrees, with accuracy +/- 0.5 degree. This is the range of angles the pendulum moves through, due to mechanical vibrations from coupling of the chamber to the pump. We irradiated the sail at each position with microwave power up to 30 sec.

The principal result is that the sail moved due to the microwave torque overcoming the gravitational torque, and passed through the (unstable) vertical position. The power level where this occurred appears in Figure 2 for each initial angle of deflection.

The movement of the pendulum we observed through the video camera taped and downloaded onto computer. We measured the power level corresponding to the movement of the pendulum, to within +/-50 W. The experiment divides into two separate tests:

Static Pressure Balancing (lower curve): Sail stability against gravitational torques occurs at the power levels predicted. This verifies the view inspired by the UNM numerical stability studies: *power gradients of form* $\cos^m \theta$ *can stabilize sail motion if m> 4.5. In our case, m ~ 5, so our experiment agrees with this feature of the UNM simulations.*

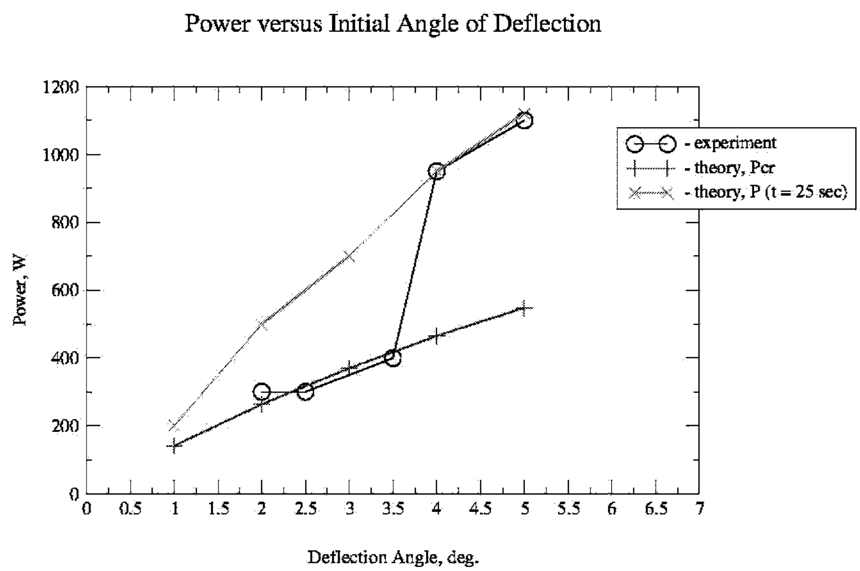


FIGURE 3. Power at Which the Sail Moved: Microwave torque overcame gravitational torque vs. initial angle of deflection. Points are data; curves are theory with no fitting parameters.

In Figure 2, the calculated value of P_{cr} ($\omega = 0$), the static case, predicts the power at which the pendulum remains at its initial angle of deflection after the pendulum is released by the actuator. The lower curve is theory for such static sail holding (with *no parameter fitting*) and fits well the power needed to just barely sustain the sail.

The calculations described in Section 2 suit the pendulum used in the experiment: pendulum off-center 1 cm from the center line of the waveguide, sail radius 2.5 cm, sail mass 18.5 mg, pendulum rod 8 cm, rod mass 397 mg, position of the center of mass of the system at the distance $\Delta L = L/50 = 0.8$ mm. The value of the center of mass (0.08-cm) is well within the adjustment capability of the countermass. Here L is the distance between sail and pivot (4 cm); distance between the sail and the end of the waveguide is 4 cm.

Dynamic Motions (upper curve): Vibrations in the system explain the transition from static force balance (lower curve of Figure 2) to the power needed to stabilize the sail against gravitational torques (upper curve of Figure 2). The lower angle cases are held in static equilibrium by the microwave power, but can be *driven by vibrations* to vertical (sail down).

At the higher angles, vibrations are insufficient to start the motion, so the full power calculated from the simple force equation must be used to move them to vertical and beyond. Therefore, the transition in Fig. 3 corresponds to the vibration-dominated limit of dynamics. The transition in Figure 3 at about 4 degrees shows that vibration accelerations can flip the sail to vertical for angles less than this, but the full power calculated for sail motion is needed at higher angles. That implies the vibrations give the pendulum accelerations of g (θ/56) =0.07 g. Reducing vibrations should then show a transition at lower angles.

CONCLUSIONS

The basic equations above are compatible with our static force measurements. Observing oscillatory measurements will demand a higher power level in future experiments. This implies that *achieving stability demands a sufficient gradient of microwave power across the sail diameter.* This is why placing the sail slightly off center from the microwave beam greatly aids stabilizing motion, since there the power gradient is larger. *This feature will be true of actual flying sails as well.*

Note that we have not studied the influence of *sail shape* on stability. The UNM simulation results to date say that a shallow "Chinese hat" sail is stable, but no other shape has yet. The class of unstable sails includes the simple flat sail we used in the pendulum experiments, which in the UNM simulations slides readily down the power gradient in angle. In the UNM work, spin seems unable to correct for this propensity. Further experimental work is needed to study sail shape.

NOMENCLATURE

R = radius of the sail ($2R$ is a side of a square sail) (m)
ΔL = distance from the pivot to the center of mass (m)
θ = angle of microwave beam propagation (°)
ϕ_1 = pendulum decline angle from vertical position (°)
K = integral that assembles all the geometric factors of the electrodynamic pressures (m^{-1})
I = moment of inertia ($kg\text{-}m^2$)

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